Rise of Gas Bubbles in Quiescent Liquids

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The first semianalytical equation describing the rise of a single gas bubble with any size and shape through a quiescent contaminated liquid is proposed. This equation is based on two main assumptions: 1) internal bubble recirculation has no effect on the velocity of bubble rise, and therefore the drag coefficient of the rising bubble equals that of a rising light solid particle (solid bubble); 2) the drag coefficient of the bubble can be calculated on the basis of its real geometric characteristics. A good fit between the velocities predicted from this equation and the published experimental data was observed. The latter was obtained from 32 gas-liquid systems with liquid densities between 0.79 and 13.5 g/cm^3, liquid viscosities from 0.80 to 120 cp, and surface tensions of 20 to 478 dyne/cm.

Gas-liquid processes in which gas bubbles rise through a liquid are very popular in the engineering practice. They are very important to chemical engineering (Froment and Bishoff, 1979), metallurgy, and especially in biotechnology (Bailey and Ollis, 1977; Cooney, 1983; Nikolov and Karamanev, 1987). The knowledge of the fundamentals of hydrodynamics, particularly the velocity of free rise of a single gas bubble in liquid media, is of great importance to understanding the characteristics of such processes.

The rise of gas bubbles has been studied since the beginning of this century (Allen, 1900). Many different equations for the calculation of the rising velocity of gas bubbles have been proposed. Equations based on the model of a bubble with internal circulation (Hadamard, 1911; Rybczynsky, 1911; Levich, 1962) often fail to adequately describe real systems (Clift et al., 1978), mainly because even highly purified liquids (such as triple distilled water) contain enough surface-active components to affect internal bubble recirculation. The most reliable semiempirical equation is that of Davies and Taylor (1950):

$$U = 25\ V^{1/6} \tag{1}$$

where V is the bubble volume. This equation, however, works only in the case of large, spherical cap-shaped bubbles. Among the empirical correlations, that of Grace et al. (1976) covers the broadest range of experimental data, but unfortunately it is quite complicated. In general, there is no single equation, describing the bubble rising velocity for the entire region of Reynolds numbers, and therefore bubble sizes and shapes.

The drag coefficient C_D of the gas bubble is calculated by most authors on the basis of the equivalent sphere diameter. A very large deviation of C_D as a function of Re is observed when different liquids are used. Only a few authors have used the real bubble shape for calculation of C_D (Davies and Taylor, 1950; Calderbank and Lohiel, 1964; Raymond and Zieminski, 1971; Miyahara and Takahashi, 1985). Some of them reported that the scatter of the drag coefficient decreased significantly. Unfortunately, these studies were performed in narrow ranges of Re or within a limited range of the physicochemical parameters.

The rising velocity and the drag coefficient of a bubble in liquid were compared to these of a free-falling heavy particle (Levich, 1964; Gaudin, 1957). This approach is based on the presumption that free-falling heavy spheres behave exactly like freely rising solid light spheres (except for the direction of the speed) since the same forces, but with opposite directions, are applied. This assumption was found to be incorrect by Karamanev and Nikolov (1992), especially for particles with densities below 0.3 g/cm^3 and $Re > 130$ rising in water. In this region of parameters, the drag coefficient is constant, equal to 0.95 (Karamanev and Nikolov, 1992). It is more than twice that predicted by Newton's law (Newton, 1760) for falling particles. The light spheres always rise by a spiral trajectory in the abovementioned range of parameters. The trajectory of rising meteorological balloons is similar to that of a rising solid sphere in liquid (Karamanev and Nikolov, 1992). Moreover, the trajectory of rising spherical and ellipsoidal gas bubbles at higher Reynolds numbers is also identical (Tsuge and Hibino, 1971). It shows that similarities can be expected in the behavior of gas bubbles and light solid particles.

The main aim of this work is to compare the drag curve of rising bubbles (calculated on the basis of the real bubble geometry) with that of a free-rising solid sphere. A semianalytical equation for prediction of the bubble rising velocity for the entire region of Re is proposed on this basis. A large quantity of published experimental data is used to prove the validity of the equation. Experiments with "solid bubbles" were performed to provide further proof of the equation proposed.

Drag Curves of Gas Bubbles and Solid Particles

Figure 1 shows the standard drag curve, valid for the case of free-falling heavy spheres, the drag curve of free-rising light

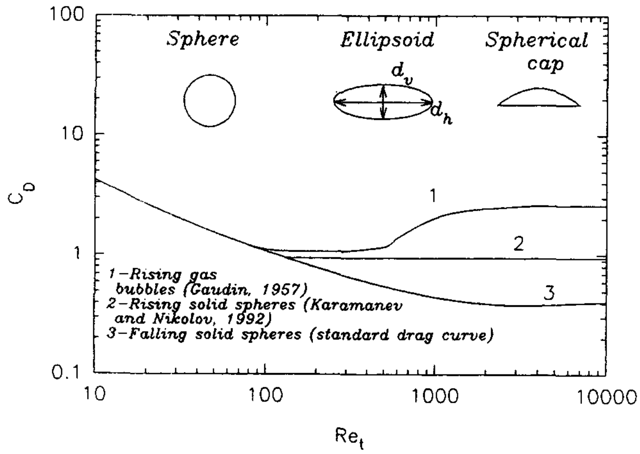

Figure 1. Comparison between drag curves.

The drag coefficient of bubbles was calculated using the equivalent sphere diameter.

spheres and the curve for rising air bubbles in contaminated water. The drag curve for gas bubbles in water is close to that for rising solid spheres up to *Re* 500. This Reynolds number corresponds to the transition from spherical to ellipsoidal bubble shape. The drag coefficient of air bubbles (C_D') rises with increasing *Re* until $Re = 4{,}500$. It remains constant and equals 2.6 when $Re > 4{,}500$. This value of *Re* corresponds to the transition from ellipsoidal shape to spherical cap. Therefore, it appears that there is a strong dependence between the shape of the bubble and its drag coefficient.

Table 1. Sources of the Data Used in Figures 2 and 4*

Liquid	Reference
water	Tsuge and Hibino (1971)
25% glycerine (aq. sol.)	″
50% glycerine (aq. sol.)	″
0.15% carboxymetilcellulose	″
0.01% laurilsulfate (aq. sol.)	″
53% glycerine (aq. sol.)	Akehata et al. (1967)
66% glycerine (aq. sol.)	″
79.5% glycerine (aq. sol.)	″
acetic acid	″
water	Davies and Taylor (1950)
nitrobenzene	″
90% glycerol (aq. sol.)**	Calderbank et al. (1970)
water**	Calderbank and Lohiel (1964)
n-heptanol	Raymond and Zieminski (1971)
4-octanol	″
n-octanol	″
4-heptanol	″
n-nonanol	″
diethyleneglycol	Uno and Kintner (1956)
0.2% trimethylnonil ether	″
water	″
water	Harmathy (1960)
water	Rosenberg (1950)
water	Haberman and Morton (1953)
22 mg/L Terpineol (aq. sol.)	Gaudin (1957)
3.9% polyvinylalcohol	Davenport et al. (1967a)
5.4% polyvinylalcohol	″
water	″
ethyl alcohol†	″
0.5% polyvinyl alcohol†	″
mercury†	Davenport et al. (1967b)
liquid-solid fluidized bed	El-Temtamy and Epstein (1980)

*The gas is always air, unless otherwise indicated.
**Carbon dioxide.
†Nitrogen.

In the following discussion, we will assume that the difference between the drag coefficients C_D given in the literature and that of a light solid particle is only due to the use of equivalent sphere diameter as measurement of the bubble size. The balance of the forces acting on a rising bubble can be written as:

$$\frac{1}{2} C_D S \rho U^2 = \Delta \rho g V \tag{2}$$

Where ρ is the liquid density, $\Delta\rho$ is the difference between the liquid and gas densities, and g is the acceleration due to gravity. To obtain the value of C_D based on the real bubble geometry, the area S of the bubble should be determined from the diameter projected on the horizontal plane circle, d_h: $S = \pi d_h^2/4$. The volume of the bubble can be calculated using the equivalent diameter: $V = \pi d_e^3/6$. Now, Eq. 2 can be solved in terms of C_D:

$$C_D = \frac{4 g \Delta\rho d_e^3}{3 \rho d_h^2 U^2} \tag{3}$$

The above equation was used to calculate the drag coefficients from the values of U available in the literature. The parameters of 32 gas-liquid systems (Table 1) used were within the following ranges: $0.79 < \rho < 13.5$ g/cm^3; $0.8 < \mu < 120$ cp; surface tension $20 < \sigma < 487$ dyne/cm; and $3.6 \times 10^{-14} < M < 3 \times 10^{-2}$. Here, M is the Morton number; $M = g\mu^4/\rho\sigma^3$. Data for Newtonian liquids and liquid-solid fluidized beds were used. All the data found in the literature were included in the analysis except: 1) it was uncertain whether the liquid was contaminated (a simple criterion is proposed by Clift et al. (1978), in which the velocity-bubble volume curve should not pass through a maximum); 2) if the liquid viscosity μ was above 120 cp at $Re < 200$. In this case, the internal recirculation of the bubble is significant. The experimental aspect ratio (d_e/d_h) was applied when available; otherwise, the Tadaki and Maeda (1961) correlation was used for determination of this parameter. When the drag coefficient was given in the literature as C_D' (calculated from d_e), its value was recalculated as: $C_D = C_D' (d_e/d_h)^2$. The data obtained are plotted in Figure 2. A very good agreement between the bubble drag and that of solid particles can be observed (standard deviation 9%) in the entire region of *Re* from 0 to 10^4 (the data for $Re < 10$ are not shown) and bubble shapes from sphere through ellipsoid to spherical cap. The average C_D for $Re > 130$ equals 0.988.

Experiments with "Solid Bubbles"

The assumption of the equality of the bubble and solid particle drag coefficients was double-checked by the following experiments. Several "solid bubbles," styrofoam particles (density of 0.030 g/cm^3) with the shapes of bubbles, were produced. The shapes and aspect ratios were determined from the Tadaki and Maeda (1961) correlation for the case of air bubbles rising in water. The drag coefficients were determined by a photographic method described earlier (Karamanev and

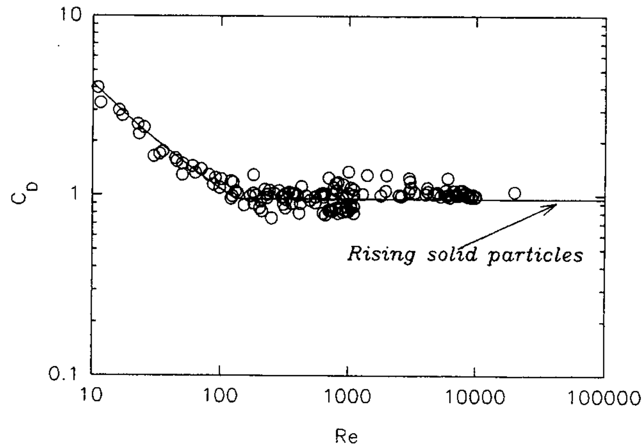

Figure 2. Drag coefficients of bubbles rising in different liquids.

The sources of data are shown in Table 1.

Nikolov, 1992). The results are shown in Figure 3. Both the ellipsoidal and spherical capped "solid bubbles" have a drag coefficient close to that for rising solid spheres (0.95). The Reynolds numbers varied between 500 and 11,000. All the ellipsoidal "solid bubbles" rose by spiral trajectory, while the spherical capped ones had rectilinear trajectories. Nevertheless, their drag coefficients were still similar (Figure 3). The same trajectories have been observed in the case of real gas bubbles.

A very important conclusion can be made from the above findings. Since all the experimental drag coefficients of rising gas bubbles agree closely with those for rising "solid bubbles," the gas bubble acts as a solid particle in the entire region of Re. Therefore, the internal recirculation has no detectable effect on dynamics of the bubble rise in contaminated liquids. This result is of great interest for modeling the dynamics of bubble rise.

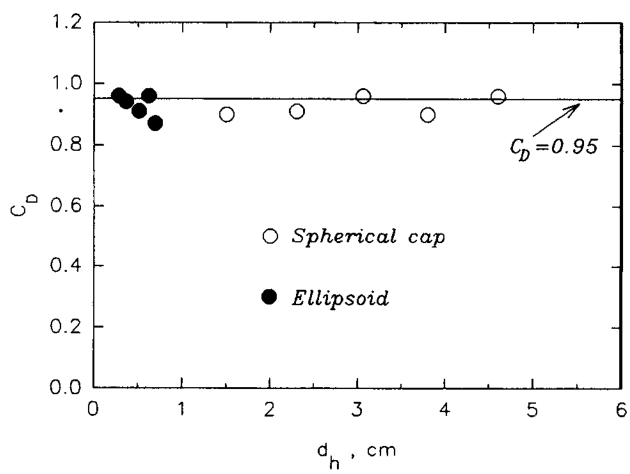

Figure 3. Drag coefficients of ellipsoidal and spherical cap "solid bubbles."

Semianalytical Equation for Calculating the Bubble Rising Velocity

Now, it is possible to obtain a semianalytical equation linking the bubble rising velocity and its geometry. Equation 3 can be rewritten in terms of U:

$$U=\sqrt{\frac{8gV}{\pi C_D d_h^2}} \tag{4}$$

After a simple geometric analysis, Eq. 4 can be rewritten as:

$$U=\sqrt{\frac{8g}{6^{2/3}\pi^{1/3}C_D}}\,V^{1/6}\,\frac{d_e}{d_h} \tag{5}$$

Since the Tadaki and Maeda (1961) correlation is among the most reliable ones for determining bubble shape, the term (d_e/d_h) in Eq. 5 can be replaced by this correlation:

$$U=\sqrt{\frac{8g}{6^{2/3}\pi^{1/3}C_D}}\,V^{1/6}\,a\cdot Ta^b \tag{6}$$

where $Ta=ReM^{0.23}$; $a=1$, $b=0$ when $Ta<2$ (spherical bubble); $a=1.14$, $b=-0.176$ when $2<Ta<6$ (ellipsoidal bubble); $a=1.36$, $b=-0.28$ when $6<Ta<16.5$ (ellipsoidal bubble); and $a=0.62$, $b=0$ when $Ta>16.5$ (spherical cap bubble). In Eq. 6, C_D can be calculated using the correlations of the drag curve for light particles (Karamanev and Nikolov, 1992):

$$C_D=\frac{24(1+0.173Re^{0.657})}{Re}+\frac{0.413}{1+16{,}300Re^{-1.09}} \tag{6a}$$

for $Re<130$ and

$$C_D=0.95 \tag{6b}$$

for $Re>130$. Alternatively, any correlation of the standard drag curve can be used instead of that of Turton and Levenspiel (1986) (Eq. 6a).

In engineering practice, bubble Reynolds numbers larger than 130 are usual. Since in this region $C_D^{1/2}\approx 1$ and $g=981$ cm/s, Eq. 6 becomes:

$$U=40.3\,V^{1/6}\,\frac{d_e}{d_h}=40.3\,V^{1/6}\,a\cdot Ta^b \tag{7}$$

An analysis of Eq. 5 with Eqs. 6a and 6b can be made for the cases of both spherical bubbles at $Re<1$ and spherical cap bubbles. In the first case, $C_D=24/Re$, $d_e/d_h=1$, and Eq. 5 transforms to the Stokes equation. It has been proved many times that in this region the gas bubble rise can be adequately described by the Stokes law.

When the bubble has a spherical cap shape, it has long been established (Clift et al., 1978; Davies and Taylor, 1950; Tadaki and Maeda, 1961) that $d_e/d_h=0.62$. The spherical cap bubbles usually have $Re>130$, and therefore Eq. 5 transforms to:

$$U=25\cdot V^{1/6} \tag{8}$$

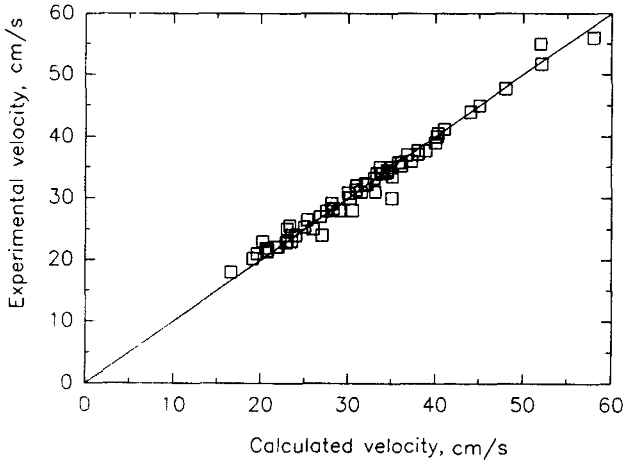

Figure 4. Experimental vs. calculated (Eqs. 6) terminal bubble velocities.

which is exactly the famous Davies-Taylor equation.

The experimental data from the literature and calculated from Eq. 6 bubble rise velocities are compared in Figure 4. A very good agreement can be observed.

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